



Class 11 Maths || Chapter 7 : Permutations and Combinations ||

Class Notes

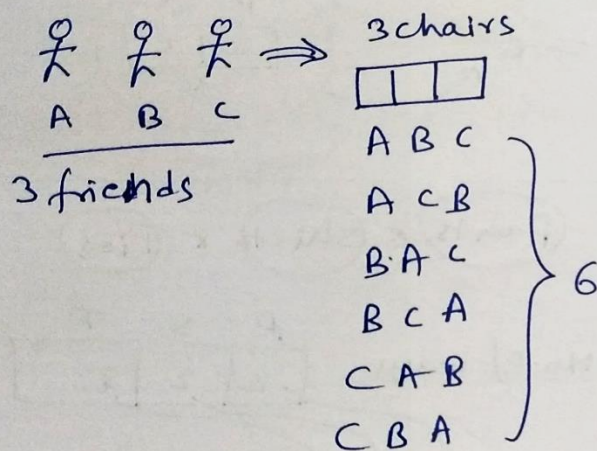
Toppers Village

Class - 11 - Maths

Chapter. 7

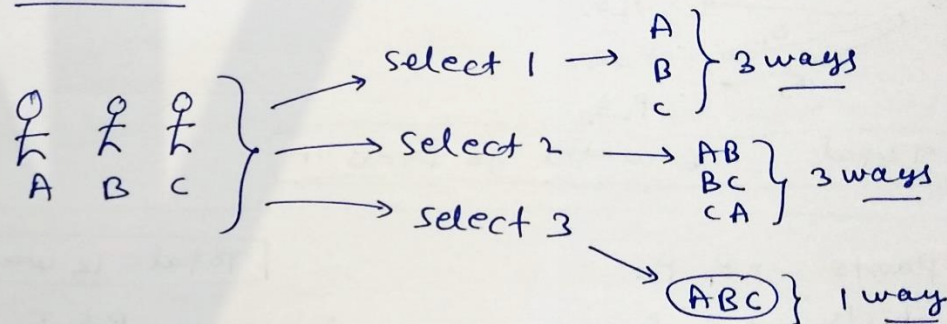
Permutations and Combinations

No. of Arrangements



Discussed in Exercise
7.1, 7.2, 7.3

No. of Selections

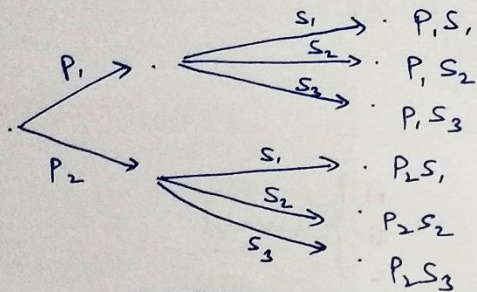


Discussed in Exercise 7.4

YouTube पर इस Chapter के FREE Videos देखिए (Playlist की Link नीचे है)

https://www.youtube.com/watch?v=oq5D1saxyV8&list=PLPFrn0ppwwkQ26oM_iKqXr_kCXy6p2HFn

e.g. ① 2 pants $\rightarrow P_1, P_2$
 3 shirts $\rightarrow S_1, S_2, S_3$



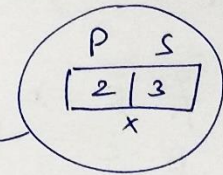
Total = 6 ways to Dress up

Solution by
Fundamental Principle of Counting
 (multiplication rule)

$$= \text{Pants} \times \text{Shirts}$$

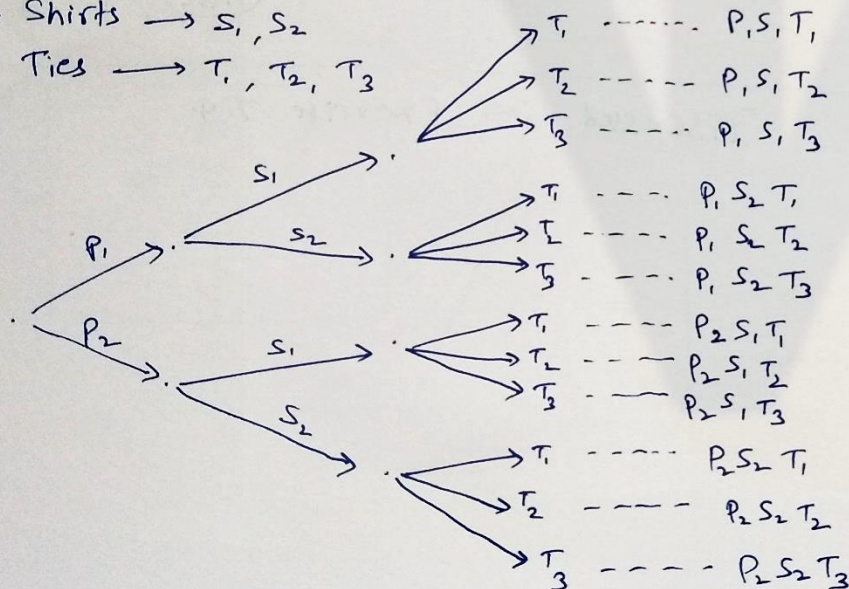
$$= 2 \times 3$$

$$= 6$$

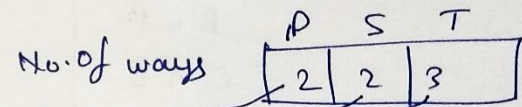


e.g. ② 2 pants $\rightarrow P_1, P_2$
 2 Shirts $\rightarrow S_1, S_2$
 3 Ties $\rightarrow T_1, T_2, T_3$

Total = 12 ways



Pants $\times$ Shirts $\times$ Ties



No. of ways

$$= 2 \times 2 \times 3$$

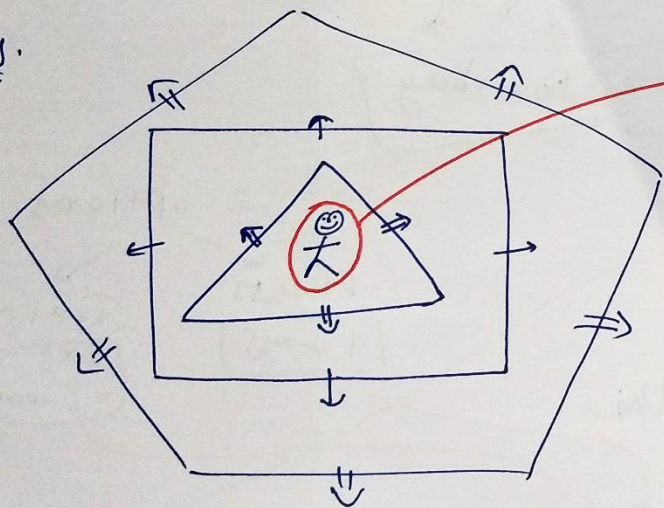
$$= 12$$

Fundamental Principle of Counting:

↓
Theorem

① Multiplication Principle. Let suppose a process is to be completed in 3 steps (I followed by II followed by III), and no. of ways to complete step I is 'm', step II $\rightarrow$ 'n', step III $\rightarrow$ 'l', then no. of ways to complete the whole process is given by $= 'm \times n \times l'$.

e.g.



No. of ways to come out of this structure = ?

$$\triangle \rightarrow 3$$

$$\square \rightarrow 4$$

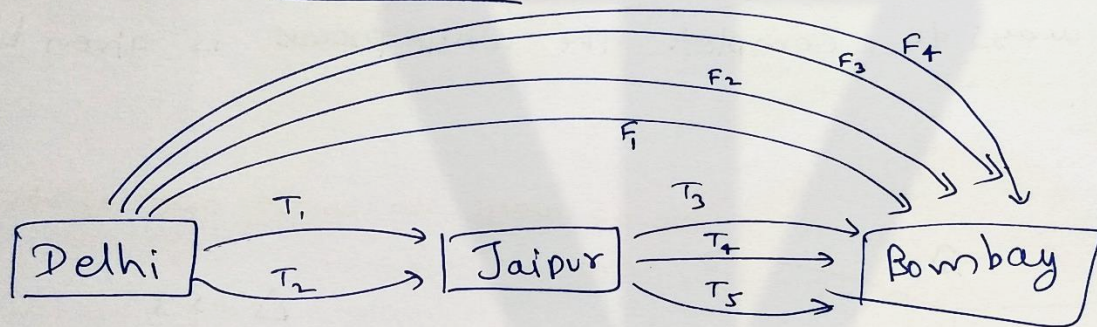
$$\pentagon \rightarrow 5$$

Total No. of in which  can come out = $3 \times 4 \times 5$

$$= 60 \checkmark$$

II Addition Principle : $\rightarrow$ If a process is to be completed by 3 options. (each option can complete the whole process)
 let No. of ways to complete I option is m , $\text{II} \rightarrow n$, $\text{III} \rightarrow l$,
 then no. of ways in which the process can be completed = $m + n + l$.

e.g.



No. of ways to reach Bombay from Delhi

$$= 4 + 6 = 10 \text{ ways}$$

2 options

Flights

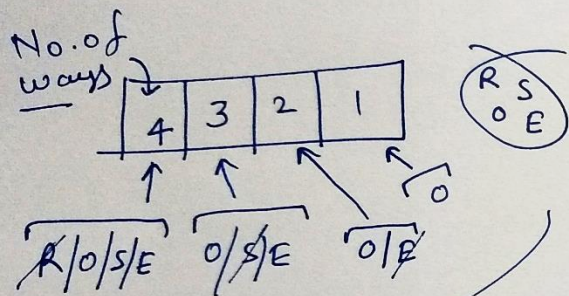
4 ways

Trains

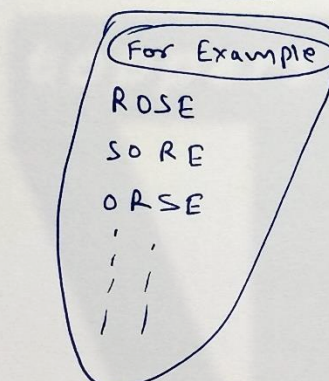
2×3
= 6 ways

e.g. No. of Different words (4 lettered) (with or without meaning)
by the letters of word "ROSE" = ?

(i) Without Repetition of Letters



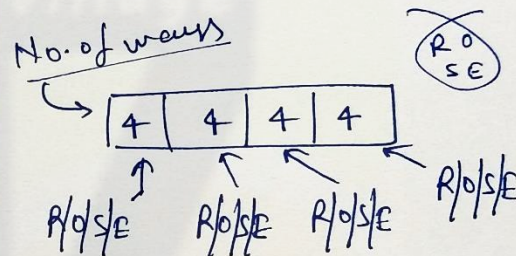
R S E O



multiplication
rule

$$= 4 \times 3 \times 2 \times 1$$
$$= 24 \text{ ways}$$

(ii) with repetition of letters



Because
Repetition
is allowed

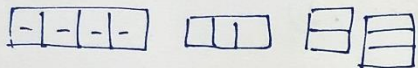
$$= 4 \times 4 \times 4 \times 4$$

$$= 256$$

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Chapter 7 Exercise 7.1 (Permutations & Combinations)

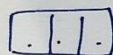
- ★ {
1. Structure
 2. Restrictions
 3. Repetition



$m+n$

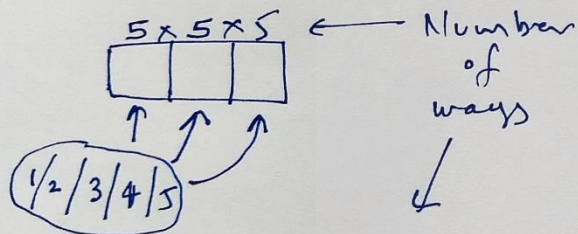
Q.1

3 Digit Numbers



Available No. = 1, 2, 3, 4, 5

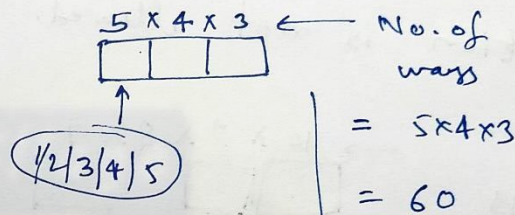
(i) Repetition Allowed.



$$= 5 \times 5 \times 5$$

$$= 125$$

(ii) Repetition is not allowed



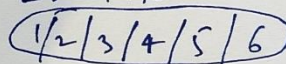
$$= 5 \times 4 \times 3$$

$$= 60$$

Q.2

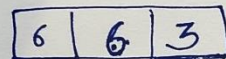
3 digit No. ✓

Even No. ✓



Repetition is Allowed

No. of ways →



$$6 \times 6 \times 3$$

$$= 108 \text{ ways.}$$

③ → "4 letter Code"

→ By First 10 letters of English Alphabet.

→ Repitition is not allowed.

↓
(a, b, c, ..., j)

No. of
ways

$$\begin{array}{c} \rightarrow 10 \times 9 \times 8 \times 7 \\ \begin{array}{|c|c|c|c|} \hline . & . & & \\ \hline \end{array} \\ \uparrow \\ \textcircled{a, b, \dots, j} \end{array} = 720 \times 7 = \underline{5040 \text{ ways}}$$

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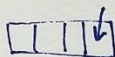
Class-II-Maths

Exercise 7.1

① Structure

② Restrictions

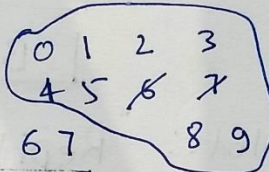
③ Repetition



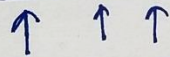
Q.4

✓ 5 Digit Telephone No.

Using '0' to '9' →



✓ Each No. starts with 67
Repetition → Not Allowed.



$8 \times 7 \times 6$ ← No. of ways

$$\begin{aligned}\text{Total No. of ways} &= 8 \times 7 \times 6 \\ &= 56 \times 6 \\ &= 336 \checkmark\end{aligned}$$

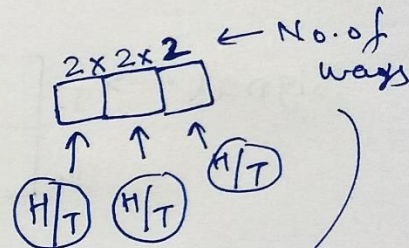
Q.5

A coin is tossed 3 times



Outcomes are recorded

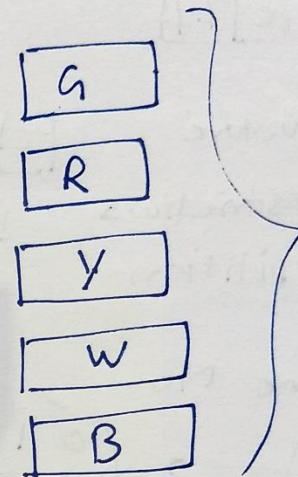
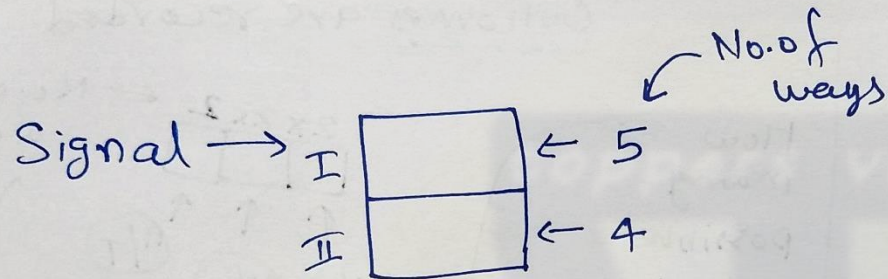
How many possible outcomes are there?



$$\begin{aligned}&= 2 \times 2 \times 2 \\ &= 8\end{aligned}$$

[Q.6]

5 flags of Different Colours →



Total No. of Signals

$$= 5 \times 4$$

$$= 20 \checkmark$$

Note: "Repetition is not"
Allowed

Self understood

Toppers Village.

Class - 11 - Maths Chapter 7

Factorial $\times$ Exercise 7.2

$$\text{Factorial of '5'} = 5 \times 4 \times 3 \times 2 \times 1 = 5! = \underline{5}$$

$$\text{Factorial of '7'} = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 7! = \underline{7}$$

$$n! = \underline{n}$$

Definition of factorial of 'n' ($n \in \mathbb{N}$)

= the product of first n natural numbers.

$$= \underline{n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdot \dots \cdot (2)(1)}$$

Note

$$\boxed{\text{Factorial of '0'} = 0! = \underline{0} = 1}$$

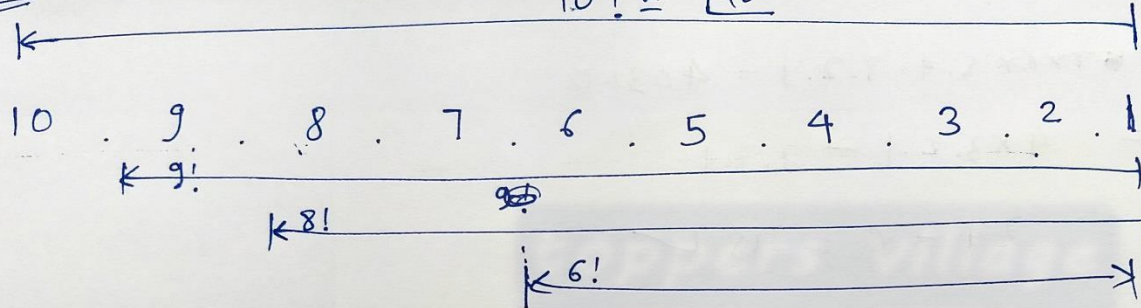
$$\rightarrow 0! = 1$$

$$N \left\{ \begin{array}{l} 1! = 1 \\ 2! = 2 \times 1 = 2 \\ 3! = 3 \times 2 \times 1 = 6 \end{array} \right.$$

$(-2)! \times$
 $(\frac{1}{2})! \times$ $\rightarrow$ Not Defined

e.g.

$$10! = \underline{10}$$



$10! = 10 \times 9 \times \boxed{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$

$$10! = \underline{10 \times 9 \times 8!}$$

Property of Factorial.

$$n! = n \times (n-1)!$$

$$= n \times (n-1) \times (n-2) \dots$$

$$= n \times (n-1) \times (n-2) \times (n-3)!$$

•
•
•
•

$$= \frac{n(n-1)(n-2)(n-3) \dots 3 \cdot 2 \cdot 1}{n!}$$

$$\begin{aligned} & \frac{8!}{4!} \\ &= \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1} \\ &= 8 \times 7 \times 6 \times 5 \\ &= 1680 \end{aligned}$$

Don't's

$$8! + 7! \neq (8+7)! = 15! \quad \times$$

$$\frac{8!}{4!} = 2!$$

Exercise 7.2

Q.1

$$(i) 8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40320$$

$$(ii) 4! - 3! = 4 \times 3 \times 2 \times 1 - 3 \times 2 \times 1 \\ = 24 - 6 = 18 \checkmark$$

Q.2

~~Is~~ Is $\underline{3! + 4! = 7!}$? $\longrightarrow$ wrong

$$\text{LHS} = 3! + 4! = \underline{3 \times 2 \times 1} + \underline{4 \times 3 \times 2 \times 1} = 6 + 24 = 30$$

$$\text{RHS} = 7! = \underline{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} = 5040$$

Q.3

Compute.

$$\frac{8!}{6! \times (2!)} = \frac{\cancel{8} \times \cancel{7} \times \cancel{6!}}{\cancel{6!} \times (2 \times 1)} = 28 \checkmark$$

④ If $\frac{1}{6!} + \frac{1}{7!} = \frac{n}{8!}$, find (n) .

$$\Rightarrow \frac{1}{6!} + \frac{1}{7!} = \frac{n}{8!}$$

$$\Rightarrow \frac{8!}{6!} + \frac{8!}{7!} = (n)$$

$$\Rightarrow n = \frac{8 \times 7 \times 6!}{6!} + \frac{8 \times 7!}{7!} = 56 + 8 = 64 \checkmark$$

⑤ Evaluate $\frac{n!}{(n-r)!}$

(i) $n=6, r=2$

$$\frac{n!}{(n-r)!} = \frac{6!}{(6-2)!}$$

$$= \frac{6!}{4!} = \frac{6 \cdot 5 \cdot \cancel{4!}}{\cancel{4!}} = 30$$

(ii) $n=9, r=5$

$$\frac{n!}{(n-r)!} = \frac{9!}{(9-5)!} = \frac{9!}{4!}$$

$$= \frac{9 \times 8 \times 7 \times 6 \times 5 \times \cancel{4!}}{\cancel{4!}}$$

$$= 15120 \checkmark$$

Permutations

Arrangements

Example (16)

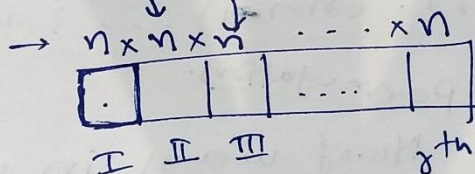
Case-I No. of Arrangements of n -objects

(all are different) at r -places

(repetition is Allowed).

CAMP

No. of ways



$$= n \times n \times n \times \dots \times n$$

(r -times)

$$= n^r$$

e.g.

CAMP

4.4.4.4 = $4 \times 4 \times 4 \times 4$ ← 4 places

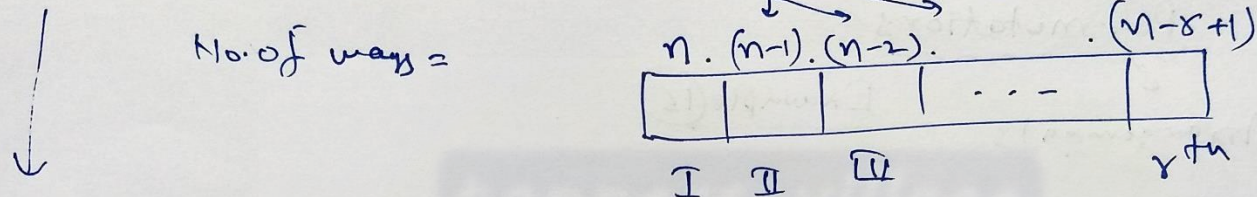
4.4.4 = $4 \times 4 \times 4$ ← 3 places

No. of ways = $4^4 = 256$

No. of ways = $4^3 = 64$

Repetition is Allowed

Case-2 No. of Arrangements of n -objects (all are different) at r -places (repetition is not allowed)



$$= n \times (n-1) \times (n-2) \times \dots \times (n-r+1)$$

$$= \frac{n \times (n-1) \times (n-2) \times \dots \times (n-r+1) \cdot \boxed{(n-r) \cdot (n-r-1) \cdot \dots \cdot 3 \cdot 2 \cdot 1}}{\boxed{(n-r) \cdot (n-r-1) \cdot \dots \cdot 3 \cdot 2 \cdot 1}}$$

$$= \frac{n!}{(n-r)!} = n P_r = \text{Permutations.}$$

No. of ways in which n -different objects can be arranged on r -places without repetition.

Objects Seats

Note $n P_0 = \frac{n!}{(n-0)!} = \frac{n!}{n!} = 1$

$$n P_1 = \frac{n!}{(n-1)!} = \frac{n \times \cancel{(n-1)!}}{\cancel{(n-1)!}} = n$$

$$n P_n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n!$$

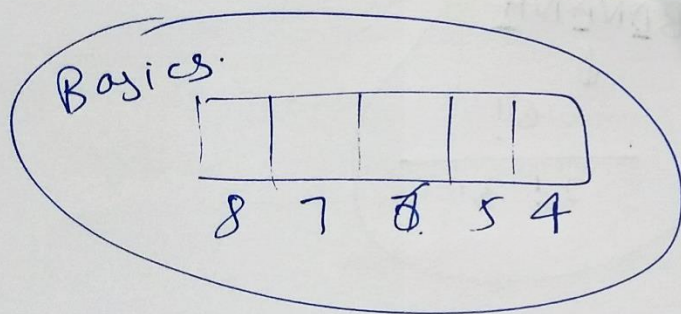
$0! = 1$

P.g. DAUGHTER $\rightarrow$ Arrange its all letters at 8 places
(i) (without repetition)

$$= {}^8P_8 = 8! = \boxed{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

DAUGHTER $\rightarrow$ (ii) Arrange its all letters at 5 places
(without repetition)

8 letters, 5 places



Permutations

$$= {}^8P_5$$

$\leftarrow$ Available candidates
 $\leftarrow$ Places.

$$= \frac{8!}{(8-5)!} = \frac{8!}{3!} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{\cancel{3 \times 2 \times 1}}$$

$$= \underline{\hspace{2cm}}$$

Case. (II) No. of Arrangements of n -objects (in which 'p' are alike of 1st kind, 'q' are alike of 2nd kind)

at n places =
$$\frac{n!}{p! q!}$$

kind = type

Reason:

$$\frac{4!}{2!}$$

same (Alike)

BOOK

$\left[\begin{array}{l} BO_1O_2K \\ BO_2O_1K \end{array} \right]$

BANANA $\rightarrow$ total = 6

B, NN, AAA

2

3

BANANA

↓

6!

2! 3!

Example 16

Independence

total = 12

I
NNN
DD
EEEE
P
C

③
②
④

e.g.
NNN IEEEE DDPC

BAG method

Total No. of Arrangements (i) words start with 'P'

$$= \frac{12!}{3! 2! 4!}$$

Fixed
11-Places
 $= \frac{11!}{3! 2! 4!}$

All the vowels never (iii) occur together

= total - those cases in which all vowels occur together

$$= \frac{12!}{3! 2! 4!} - \frac{8! \times 5!}{3! 2! 4!}$$

(iv) words begin with 'I' & end in 'P'

Fixed
10 Places

$$= \frac{10!}{3! 2! 4!}$$

All the vowels always (ii) occur together

Consonants: NNN, DD, P, C
vowels: I, EEEE

Consider as a one object

(I) → Permute NNN DD P C

$$\frac{8!}{3! 2!}$$

(II) In the Bag → Permute I EEEE

$$\text{Total} = \frac{8!}{3! 2!} \times \frac{5!}{4!}$$

Toppers Village

Class-11-Maths Exercise-7.3 (P & c)

1. Structure

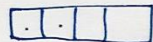
2. Restriction

Basic

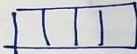
$$\frac{n!}{(n-r)!} = n P_r$$

Available Candidates (upper suffix)
seats (lower suffix)

FIRE



$$4 \cdot 3 \cdot 2 \cdot 1 = 4! \\ \rightarrow = 24$$



$${}_4P_4 = \frac{4!}{(4-4)!} = \frac{4!}{0!} \\ = \frac{24}{1}$$

Q.2

4 Digit No.

?

Available

0, 1, 2, 3, 4, 5

6, 7, 8, 9,

No Repetition



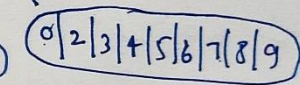
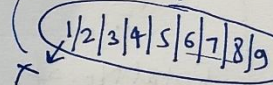
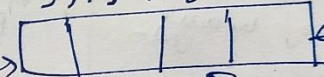
x '0'

x 05+3 → 3 digit

Indirect Restriction

No. of ways

$$\rightarrow 9 \times 9 \times 8 \times 7$$



Total No. of ways = $9 \times 9 \times 8 \times 7$

$$= 81 \times 56$$

$$= 4536$$

$$\begin{array}{r} 81 \\ \times 56 \\ \hline 486 \\ 405 \times \\ \hline 4536 \end{array}$$

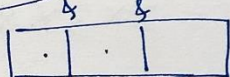
Q.1

3-digit No.

Available

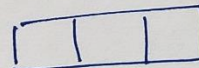
1, 2, 3, 4, 5, 6, 7, 8, 9

By Basics



$$\text{No. of ways} \rightarrow 9 \times 8 \times 7 \\ = 9 \times 56 \\ = 504$$

nPr



$$= {}_9P_3 = \frac{9!}{6!} = \frac{9 \cdot 8 \cdot 7 \cdot 6!}{6!} \\ = 9 \times 8 \times 7 = 504$$

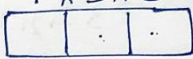
Q.3 3 digit No.

Even No.

Restriction

No. of ways

$\rightarrow 4 \times 5 \times 3$



2/4/6

$$\begin{aligned} \text{Total No. of ways} &= 4 \times 5 \times 3 \\ &= 60 \checkmark \end{aligned}$$

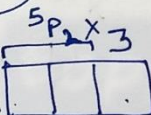
Basics.

Available

1, 2, 3, 4, 6, 7

No Repetition

nPr



nPr 2/4/6

$$= {}^5P_2$$

$$\text{Total No. of ways} = {}^5P_2 \times 3$$

$$= \frac{5!}{3!} \times 3$$

$$= \frac{5 \times 4 \times \cancel{3!}}{\cancel{3!}} \times 3$$

$$= 60$$

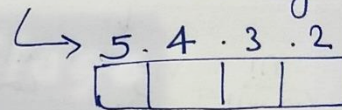
Q.4 4 digit No.

Available

1, 2, 3, 4, 5

No Repetition

(i) Total No. of 4 Digit No.



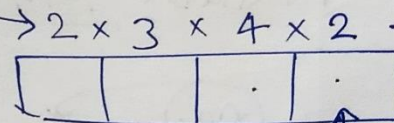
No Restriction

1/2/3/4/5

$$\begin{aligned} &= 5 \times 4 \times 3 \times 2 \\ &= 120 \checkmark \end{aligned}$$

(ii) Total No. of Even No. (4 Digit.)

RESTRICTION



2/4

$$\begin{aligned} &= 2 \times 3 \times 4 \times 2 \\ &= 48 \end{aligned}$$

Toppers Village

class-11-maths Exercise 7.3 (P&C)

Q.5 Available $\rightarrow$ 8 persons

chairman ?

Vice chairman ?



8
7
↓

and $\rightarrow$ 'x'

multiply

$$\begin{aligned}\text{Total No. of ways} &= 8 \times 7 \\ &= 56\end{aligned}$$

Q.6 Find n, if ${}^{n-1}P_3 : {}^nP_4 = 1:9$

$$\Rightarrow {}^nP_r = \frac{n!}{(n-r)!} \Rightarrow n! = n \cdot (n-1)! = n \cdot (n-1) \cdot (n-2)! = \dots$$

$$\begin{aligned}\text{Given } \frac{{}^{n-1}P_3}{{}^nP_4} &= \frac{1}{9} \Rightarrow \frac{(n-1)!}{(n-1-3)!} \cdot \frac{n!}{(n-4)!} = \frac{1}{9}\end{aligned}$$

$$\Rightarrow \frac{(n-1)!}{(n-4)!} \cdot \frac{n!}{(n-4)!} = \frac{1}{9}$$

$$\frac{1}{9} = \frac{1}{9}$$

$$\Rightarrow \frac{(n-1)!}{n!} = \frac{1}{9}$$

$$\Rightarrow \frac{(n-1)!}{n \cdot (n-1)!} = \frac{1}{9}$$

$$\Rightarrow \frac{1}{n} = \frac{1}{9}$$

$$\Rightarrow \boxed{n=9} \checkmark$$

Q.7 Find 'x'.

$$\boxed{{}^n P_r} \quad 0 \leq r \leq n$$

(i) $5P_x = 2 \cdot {}^6 P_{x-1}$

$$\Rightarrow \frac{5!}{(5-x)!} = 2 \cdot \frac{6!}{[6-(x-1)]!}$$

$$\Rightarrow \frac{5!}{(5-x)!} = 2 \times \frac{6!}{(7-x)!}$$

$$\Rightarrow \frac{\cancel{5!}}{(5-x)!} = 2 \times \frac{6 \times \cancel{5!}}{(7-x) \cdot (6-x) \cdot \cancel{(5-x)!}}$$

$$\Rightarrow (7-x) \cdot (6-x) = 12$$

$$\Rightarrow \cancel{4^2} - 13x + x^2 = 12$$

$$\Rightarrow x^2 - 13x + 30 = 0$$

$$\Rightarrow \underline{x^2 - 10x} - \underline{3x + 30} = 0$$

$$\Rightarrow x(x-10) - 3(x-10) = 0$$

$$\Rightarrow (x-3)(x-10) = 0$$

$$\boxed{x = 3, 10}$$

$x=3$ $5P_3 = 2 \times {}^6 P_{3-1}$ ✓
 $x=10$ $5P_{10} = 2 \times {}^6 P_9$ ✗
 $(10 > 5)$ ✗

(ii) $5P_x = {}^6 P_{x-1}$

$$\Rightarrow \frac{5!}{(5-x)!} = \frac{6!}{[6-(x-1)]!}$$

$$\Rightarrow \frac{\cancel{5!}}{(5-x)!} = \frac{6 \times \cancel{5!}}{(7-x) \cdot (6-x) \cdot \cancel{(5-x)!}}$$

$$\Rightarrow (7-x)(6-x) = 6$$

$$\Rightarrow 42 - 13x + x^2 = 6$$

$$\Rightarrow x^2 - \underline{13x} + 36 = 0$$

$$\Rightarrow x^2 - \underline{9x} - \underline{4x} + 36 = 0$$

$$\Rightarrow x(x-9) - 4(x-9) = 0$$

$$\Rightarrow (x-4)(x-9) = 0$$

$$\boxed{x = 4, 9}$$

$x=4$ ✓
 $5P_4 = {}^6 P_{4-1}$ ✓

$x=9$ ✗
 $5P_9 = {}^6 P_{9-1}$ ✗

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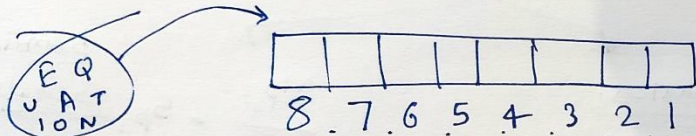
Class-11-Maths Exercise 7.3 (P&C)

Q.8 EQUATION

Available $\rightarrow$ All the letters.

Repetition $\rightarrow$ No.

$${}_nP_r = \frac{n!}{(n-r)!}$$



$$\begin{aligned}\text{By Basics} &= 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \\ &= 8 \times 5040 \\ &= 40320 \checkmark\end{aligned}$$

By ${}_nP_r$ Available candidates $= n!$
No. of seats $= (n-r)!$

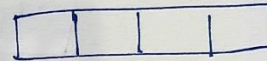
$$= {}_8P_8 = \frac{8!}{(8-8)!} = \frac{8!}{0!} = \frac{40320}{1} \checkmark$$

Q.9

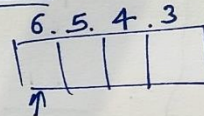
MONDAY

Repetition
 $\downarrow$
Not Allowed

(i) 4 letters at a time.



Basics.



M | O | N | D | A | Y

$$\begin{aligned}\text{Total No. of} \\ \text{words} &= 6 \cdot 5 \cdot 4 \cdot 3 \\ &= 360.\end{aligned}$$

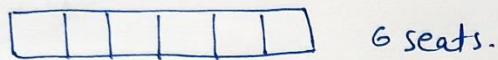


Available
 $\rightarrow$ ${}_nP_r$
 $\rightarrow$ Seats

$$\begin{aligned}&= {}_6P_4 \\ &= \frac{6!}{2!} \\ &= \frac{720}{2} \\ &= 360 \checkmark\end{aligned}$$

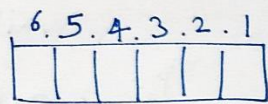
MONDAY

(ii) all letters are used at a time.



6 seats.

Basic

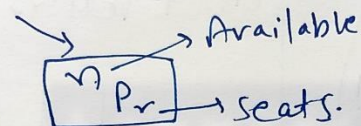


M/O/N/D/A/Y

Total No. of

$$\text{words} = 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$= 720$$



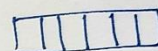
$$= {}^6P_6$$

$$= \frac{6!}{(6-6)!}$$

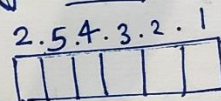
$$= \frac{6!}{0!} = \frac{720}{1}$$

(iii) all letters are used but first letter is a vowel.

Restriction ← Priority.



Basics.



⓪/A A/M/N/D/Y

No. of words

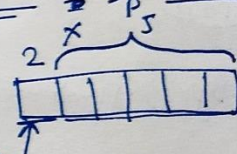
$$= 2 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

$$= 2 \times 120$$

$$= \underline{240}$$

nPr

5 candidates
5 seats



⓪/A

No. of words

$$= 2 \times {}^5P_5$$

$$= 2 \times \frac{5!}{0!}$$

$$= 2 \times \frac{120}{1} = \underline{240}$$

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Class - II - Maths Exercise 7.3 (P&C)

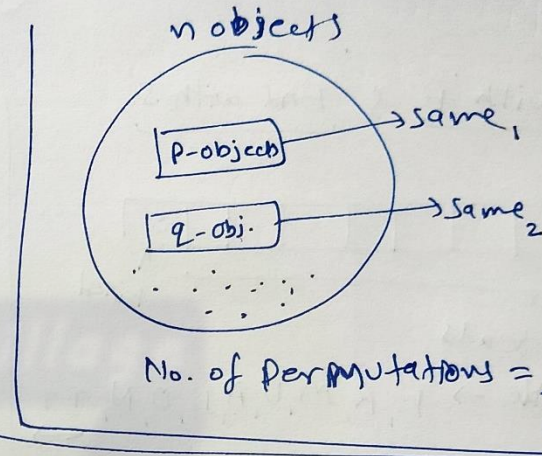
* Q.10

MISSISSIPPI $\rightarrow$ M S S S S P P
I I I I

No. of words
in which
all I's do not
occur together
?

= Total No. of words - No. of words in which all I's occur together.

$$= \frac{11!}{4! 2! 4!} - \frac{8!}{4! 2!} = \underline{\underline{\text{Answer.}}}$$



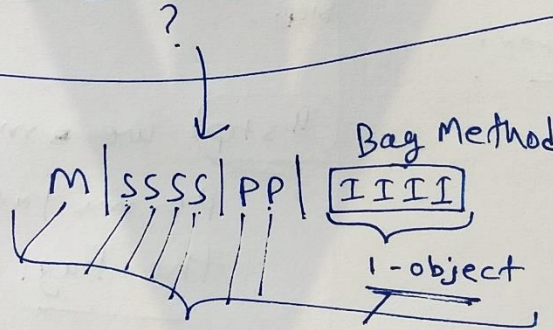
M | S S S S | P P | I I I I $\rightarrow$ (i)

= $\frac{11!}{4! 2! 4!}$ Total.

SSSS $\rightarrow$ 4!
PP $\rightarrow$ 2!
IIII $\rightarrow$ 4!

= $\frac{8!}{4! 2!}$

SSSS $\rightarrow$ 4!
PP $\rightarrow$ 2!

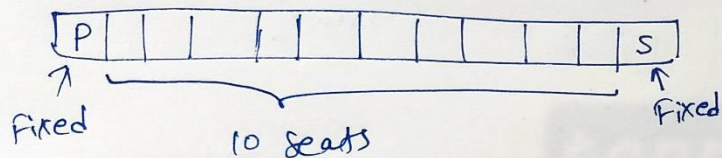


★ Example 16

Q.11 PERMUTATIONS $\rightarrow$ total = 12

★

(i) Start with 'P' & End with 'S'



Available $\rightarrow$ E, R, M, U, A, I, O, N, T, T

$$\text{No. of permutations} = \frac{10!}{2!}$$

$$= \frac{10!}{2!} = \underline{\underline{\text{Answer}}}$$

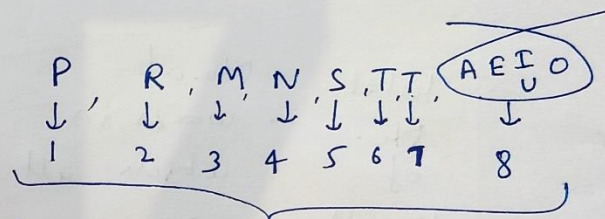
P | E | R | M | U | A | I | O | N | S | T | T

(ii) Vowels are together

BAG METHOD

~~Vowels~~ Vowels $\rightarrow$ E, U, A, I, O

Consonants $\rightarrow$ P, R, M, N, S, T, T



I Arrange these only, = $\frac{8!}{2!} \leftarrow \text{TT}$

II step we must also consider the permutations of AEIOU inside the Bag. = $\underline{\underline{5!}}$

Total No. of words in which all vowels occur together

$$= \frac{8!}{2!} \times 5! = \underline{\underline{\text{Answer}}}$$

(iii) "PERMUTATIONS"

↳ there are always 4 letters between P & S?

~~P, S~~, E, R, M, V, A, I, O, N, T, T
 10
 same

for e.g:
 - - P - - - S - - -
 (S) - - - - (P)

2 { P - - - - S
 S - - - - P

Similarity
 ↳

X	.	.	.	.	X	.	.	.	.	.	.
.	X	.	.	.	.	X	.	.	.	.	.
.	.	X	.	.	.	.	X	.	.	.	.
.	.	.	X	.	.	.	.	X	.	.	.
.	.	.	.	X	.	.	.	.	X	.	.
.	.	.	.	.	X	.	.	.	.	X	.
.	.	.	.	.	.	X	.	.	.	.	X

{ - P - - - S
 - S - - - P }

Remaining seats 10

$2 \times \frac{10!}{2!} \rightarrow (TT)$

Total No. of ways

$= 7 \times 2 \times \frac{10!}{2!} = \underline{\underline{\text{Answer}}}$

... X ... X ...

This pattern can
 Shift 7 times.

{ - P - - - S
 - S - - - P }

Remaining

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Class-11-Maths chapter 7 (P & C) Ex 7.4

COMBINATIONS	$n C_r$	EXAMPLE-19
--------------	---------	------------

Selection

Permutations: No. of permutations of n -objects at ' r ' places:
(arrangement)

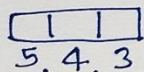
$$= {}^n P_r = \frac{n!}{(n-r)!} \quad (0 \leq r \leq n)$$

Combinations: No. of combinations of r -objects out of n -objects.
(Selection)

$$= {}^n C_r = ? \quad (0 \leq r \leq n) = \frac{n!}{(n-r)! r!}$$

e.g. 5-Persons $\rightarrow$ 3 seats (Arrange) $\rightarrow$ permutations

(i)



$$\text{No. of Permutations} = 5 \cdot 4 \cdot 3 = {}^5 P_3$$

(ii)

Select '3' out of '5' & Arrange '3' persons on 3 seats. = No. of ways

$$\hookrightarrow {}^n C_r = {}^5 C_3$$



$$= 6 = 3! \hookleftarrow$$

$$= {}^5 C_3 \times 3!$$

$$\begin{aligned} {}^5 P_3 &= ({}^5 C_3) \times 3! \\ \Rightarrow \frac{5!}{(5-3)! 3!} &= {}^5 C_3 \end{aligned}$$

Properties of $n C_r$

★ ① $n C_r = \frac{n!}{(n-r)! r!}$ ★

$$n C_r = \frac{n P_r}{r!} \Rightarrow n P_r = n C_r \cdot r!$$

n $\nearrow$ upper suffix.
 $(r$ $\rightarrow$ Combination (selection)
 $)$ $\searrow$ lower suffix.

② $n C_0 = 1$

$$n C_0 = \frac{n!}{(n-0)! 0!} = \frac{\cancel{n!}}{\cancel{n!} \times 1} = 1$$

$$n C_n = 1$$

$$n C_n = \frac{n!}{(n-n)! n!} = \frac{\cancel{n!}}{0! \times \cancel{n!}} = 1$$

$$n C_1 = n \quad \checkmark$$

$$n C_{n-1} = n \quad \checkmark$$

Properties of nCr

③ $nCr = nC_{n-r}$

e.g. ${}^{10}C_3 = {}^{10}C_{10-3}$

${}^{10}C_3 = {}^{10}C_7$ $3+7=10$

e.g. ${}^{12}C_4 = {}^{12}C_8$ $4+8=12$

LHS $= {}^nC_r = \frac{n!}{(n-r)! r!}$ ✓

RHS $= {}^nC_{n-r} = \frac{n!}{[n-(n-r)]! (n-r)!}$
 $= \frac{n!}{r! (n-r)!}$ ✓✓

④ If $nCa = nCb$ then $\begin{cases} \text{(i) } a=b \text{ or} \\ \text{(ii) } a+b=n \end{cases}$

e.g. ${}^nC_7 = {}^nC_8$
 $n=?$
 $7+8=n$
 $15=n$

Properties of nC_r

⑤ ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

${}^7C_3 + {}^7C_4 = {}^8C_4$

${}^nC_r = \frac{n!}{(n-r)! r!}$

Proof: LHS = ${}^nC_r + {}^nC_{r-1}$

$$\text{LHS} = \frac{n!}{(n-r)! \cdot r!} + \frac{n!}{(n-r+1)! (r-1)!}$$

$$= \frac{n!}{(n-r)! \cdot r \cdot (r-1)!} + \frac{n!}{(n-r+1) \cdot (n-r)! (r-1)!}$$

$$= \frac{n!}{(n-r)! (r-1)!} \cdot \left\{ \frac{1}{r} + \frac{1}{n-r+1} \right\}$$

$$= \frac{n!}{(n-r)! (r-1)!} \cdot \left\{ \frac{n-r+1 + r}{r(n-r+1)} \right\}$$

$$= \frac{(n+1)!}{(n-r+1)! r!}$$

RHS = ${}^{n+1}C_r$

$$= \frac{(n+1)!}{(n+1-r)! r!}$$

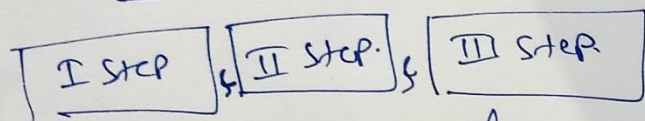
LHS = RHS

Fundamental Principle of Counting :

Before Example (19)

① Multiplication Rule (&)
(and)

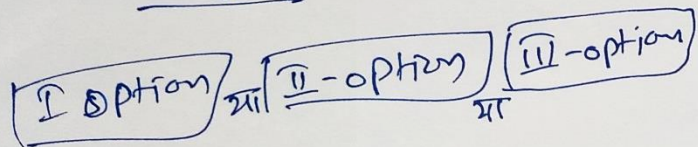
Project.



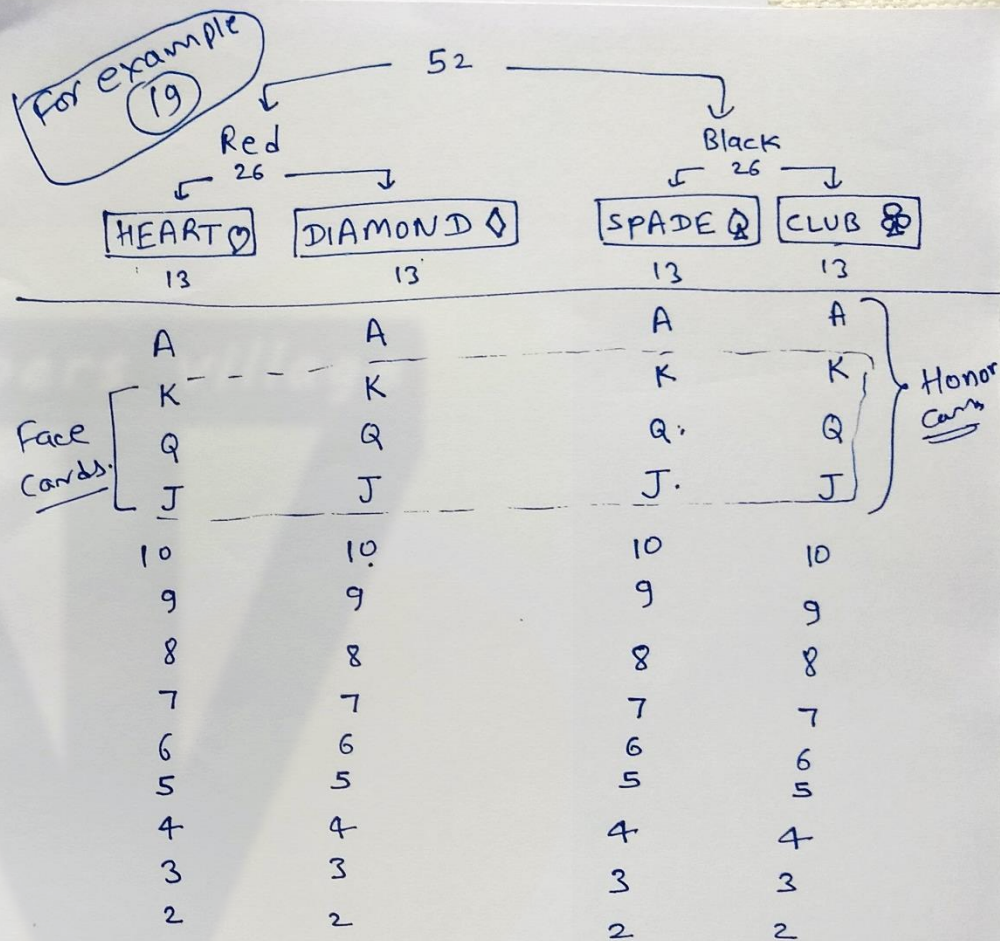
$$= m \times n \times l$$

② Addition Rule (or)
(या)

Project.



$$m + n + l$$



Example - [19] chapter (7)

No. of ways of choosing

4 cards from 52 playing cards = ${}^{52}C_4$ ✓

(i) 4 cards are of the same suit = either Heart or Dia. or Spade or club

$$= {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4$$

✓

(ii) 4 cards belong to the 4 different suits = 1 from Heart & 1 from Diamond & 1 from Spade & 1 from club

$$= {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1$$
$$= (13)^4$$

✓

(iii) are face cards $\rightarrow 12 = \text{total.}$

$$= {}^{12}C_4 = \frac{12!}{8! 4!}$$

(iv) 2 are red & 2 are black $\rightarrow 26$

$$= {}^{26}C_2 \times {}^{26}C_2$$

$$= ({}^{26}C_2)^2$$

(v) Cards are of the same Colour $\rightarrow 26$

either Red Cards or Black $\rightarrow 26$

$$= {}^{26}C_4 + {}^{26}C_4$$

$$= 2 \times {}^{26}C_4$$

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Class-11-Maths

Ex-7.4

P & C

Revision:

① $nCr = \frac{n!}{(n-r)! r!}$

② $nCr = nC_{n-r}$

$\rightarrow {}^{13}C_2 = {}^{13}C_{11}$

③ $nCa = nCb \rightarrow$
 (i) $a = b$ or
 (ii) $a + b = n$ ★

Extra Prop.

$nCr + nC_{r-1} = {}^{n+1}Cr$

Q.1

$n_{C_8} = n_{C_2}$; Find n_{C_2}

By property (3).

$8 \neq 2$

then

$8 + 2 = n$

$\Rightarrow \boxed{n = 10}$

$n_{C_2} = {}^{10}C_2$

$= \frac{10!}{8! 2!}$

$= \frac{5 \times 10 \times 9 \times 8!}{8! \times 2 \times 1} = 45$ ✓

Q.2 (i) ${}^{2n}C_3 : {}^nC_3 = 12 : 1$

★ ${}^nC_r = \frac{n!}{(n-r)! r!}$

$$\Rightarrow \frac{\left[\frac{(2n)!}{(2n-3)! 3!} \right]}{\left[\frac{n!}{(n-3)! 3!} \right]} = \frac{12}{1}$$

$$\Rightarrow \frac{\frac{(2n)(2n-1)(2n-2)(2n-3)!}{(2n-3)!}}{\frac{n \cdot (n-1)(n-2) \cdot (n-3)!}{(n-3)!}} = \frac{12}{1}$$

$$\Rightarrow \frac{4(2n-1)}{n-2} = \frac{12}{1}$$

$$\Rightarrow 2n-1 = 3n-6$$

$$\Rightarrow 6-1 = 3n-2n \Rightarrow \boxed{5=n}$$

(ii) ${}^{2n}C_3 : {}^nC_3 = 11 : 1$

$$\Rightarrow \frac{4(2n-1)}{n-2} = \frac{11}{1}$$

$$\Rightarrow 8n-4 = 11n-22$$

$$\Rightarrow 22-4 = 11n-8n$$

$$\Rightarrow 8n = 18$$

$$\boxed{n=6} \checkmark$$

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Class - 11 - Maths Exercise 7.4 (P&C)

Revision, ① n_{C_r} = No. of combinations (selection)
of r -objects taken from n -objects.

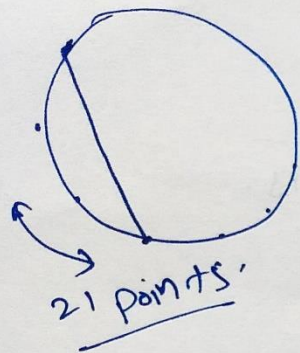
$$\textcircled{2} \quad n_{C_r} = \frac{n!}{(n-r)! r!} \quad (0 \leq r \leq n)$$

$$\textcircled{3} \quad \text{multiplication Rule } (\rightarrow \&) \quad \begin{array}{c} \boxed{\text{I Step}} \text{ \& \& } \boxed{\text{II Step}} \\ m \quad \times \quad n = m \cdot n \end{array}$$

$$\text{Addition Rule (or)} \quad \begin{array}{c} \boxed{\text{I option}} \text{ or } \boxed{\text{II - option}} \\ m \quad + \quad n = m+n \end{array}$$

Q.3

21 points on a ~~circle~~ Circle



→ one chord is made by joining two particular points of circle.

→ So we have to select 2 points to make a chord.
↓
(out of 21 points.)

No. of chords = No. of ways to select 2 points of the circle (out of 21 points.)

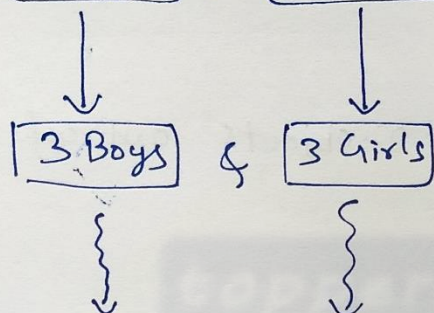
$$= {}^{21}C_2$$

$$= \frac{21!}{19! 2!} = \frac{21 \times \overset{10}{\cancel{20}} \times \cancel{19!}}{\cancel{19!} \times \cancel{2} \times 1} = 210 \checkmark$$

Q.4

from 5 Boys & 4 Girls

Select a team



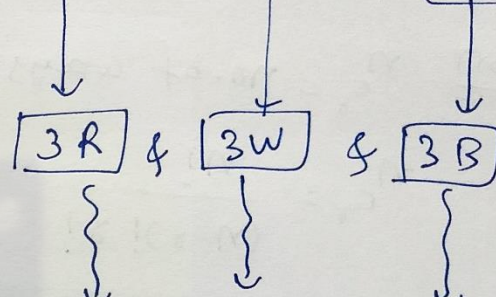
No. of ways to select a team

$$\begin{aligned} &= {}^5C_3 \times {}^4C_3 \\ &= \frac{5!}{2!3!} \times \frac{4!}{1!3!} \\ &= \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 3 \cdot 2 \cdot 1} \times \frac{4 \cdot 3 \cdot 2 \cdot 1}{1 \cdot 3 \cdot 2 \cdot 1} \\ &= 5 \times 2 \times 4 \\ &= 40 \end{aligned}$$

Q.5

from 6 Red, 5 white, 5 Blue

Select total 9-Balls



No. of ways to select 9 balls

(if 3 balls of each color)

$$\begin{aligned} &= {}^6C_3 \times {}^5C_3 \times {}^5C_3 \\ &= \frac{6!}{3!3!} \times \frac{5!}{2!3!} \times \frac{5!}{2!3!} \\ &= \frac{6 \cdot 5 \cdot 4 \cdot 3!}{3! \cdot 3!} \times 10 \times 10 \\ &= 20 \times 10 \times 10 \\ &= 2000 \checkmark \end{aligned}$$

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class - II - maths. (P & C) Exercise 7.4

Revision $nCr =$ no. of ways of selecting r -objects out of n -objects.

$$nCr = \frac{n!}{(n-r)! r!}$$

Q.6

Total 52 Cards

Select

5 cards.

4 A & 48 other cards

1 Ace & 4 other cards

Exactly

Select.

Total No.
of ways

$$= \binom{4}{1} \times$$

$\times$

$$\binom{48}{4}$$

$$= 4C_1 \times 48C_4$$

$$= 4 \times 48C_4$$

Q.7

from
total 17
players

5 Bowlers

12 others

← Available

Select Cricket
team of 11
Players

4 Bowlers

Exactly

7 others

← Desire.

Total No. of
ways of

Selecting a Cricket
team of 11-players

$$= {}^5C_4 \times {}^{12}C_7 = {}^5C_4 \cdot {}^{12}C_7$$

$$= \frac{\cancel{5!}}{1! 4!} \cdot \frac{12!}{\cancel{5!} 7!}$$

$$= \frac{12 \cdot 11 \cdot 10 \cdot \cancel{9} \cdot \cancel{8} \cdot \cancel{7}!}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1 \cdot \cancel{7}!}$$

$$= 12 \times 11 \times 10 \times 3$$

$$= 3960 \checkmark$$

8

Bag

5 Black
Balls

6 Red
Balls

No. of ways of selecting 2 Black & 3 Red Balls

$$= {}^5C_2 \times {}^6C_3$$

$$= \frac{5!}{3!2!} \times \frac{6!}{3!3!}$$

$$= \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 2 \cdot 1} \times \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3! \cdot 3 \cdot 2 \cdot 1}$$

$$= 10 \times 20 = 200$$

9

Compulsory

2 specific → Fixed by Question itself.

Available
9 Courses

2 Specific
Compulsory

+ 7 others

Choose
5 Courses

2 Specific
Compulsory

& 3 others
optional

$${}^2C_2 = 1$$

No. of
ways

$$= 1 \times {}^7C_3$$

of choosing

the programme

$$= 1 \times {}^7C_3$$

$$= 1 \times \frac{7!}{4!3!}$$

$$= \frac{7 \cdot 6 \cdot 5 \cdot 4!}{4! \cdot 3!}$$

$$= 35$$

Q.1 'DAUGHTER' Available

Vowels

Consonants

③ A, U, E

⑤ D, G, H, T, R

$$\frac{n!}{(n-r)! r!}$$

$${}^n C_r = \text{Selection}$$

Combination
(Selection)Desired

2 vowels

3 consonants.

$$\downarrow$$

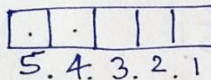
$${}^3 C_2$$

$$\downarrow$$

$${}^5 C_3$$

Repetition
↓
No.

Different words



$$= 5! = 120$$

(Arrangement)

Permutation
(Arrangement)

Total No. of words

= No. of ways of Selection (Combination) × No. of ways of Arrangement (Permutation)

$$= {}^3 C_2 \times {}^5 C_3 \times 120$$

$$= \frac{3!}{1! 2!} \times \frac{5!}{2! 3!} \times 120 = \frac{3 \times 2 \times 1}{1 \times 2 \times 1} \times \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 3 \times 2 \times 1} \times 120 = 3600$$

Q.2 'EQUATION'

All the letters

Repetition
↓
Not Allowed

Vowels Together

'EUAIO'

Bag_v

AEIOU

Consonants together

'QTN'

Bag_c

QTN

For e.g.

AEIOU QTN → ✓

EIAUO INQ → ✓

TNQ EIAUO ✓

Arrangement of Bags = 2

V C
C V

Arrangement of vowels in their own Bag = 5!

AEIOU

1 1 1 1 1
5 4 3 2 1

Arrangement of consonants in their own Bag = 3!

QTN

3 2 1
1 1 1

Total no. of words = $2 \times 5! \times 3!$

$$= 2 \times 120 \times 6 = 1440 \checkmark$$

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Miscellaneous Exercise on Chapter 7

(P & C)

$${}^nC_r = \frac{n!}{(n-r)! r!}$$

Q.3 Total Available = 9 Boys & 4 Girls
Form Committee by 7 members

(i) Exactly 3 Girls

Committee 7-member

& $\begin{matrix} 3G \\ 4B \end{matrix} \rightarrow \text{selection } {}^4C_3$
 $\begin{matrix} 3G \\ 4B \end{matrix} \rightarrow {}^9C_4$

$$= {}^4C_3 \times {}^9C_4$$

(Girls) (Boys)

$$= \frac{4!}{1!3!} \times \frac{9!}{5!4!}$$

$$= \frac{4 \cdot 3!}{1 \times 3!} \times \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{5! \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$= 4 \times \frac{9 \cdot 7 \cdot 2}{126}$$

$$= 504$$

(ii) at least 3 Girls
(अथवा 3 से अधिक) $\rightarrow 3, 4G$

$\begin{matrix} 3G \\ 4B \end{matrix}$ or $\begin{matrix} 4G \\ 3B \end{matrix}$

$$= {}^4C_3 \times {}^9C_4 + {}^4C_4 \times {}^9C_3$$

$$= \frac{4!}{1!3!} \times \frac{9!}{3!4!} + \frac{4!}{0!4!} \times \frac{9!}{6!3!}$$

Answer.

(iii) at most 3 girls

(अधिकतम 3 लड़कियाँ)

$\begin{matrix} 3G \\ 4B \end{matrix}$ or $\begin{matrix} 2G \\ 5B \end{matrix}$ or $\begin{matrix} 1G \\ 6B \end{matrix}$ or $\begin{matrix} 0G \\ 7B \end{matrix}$

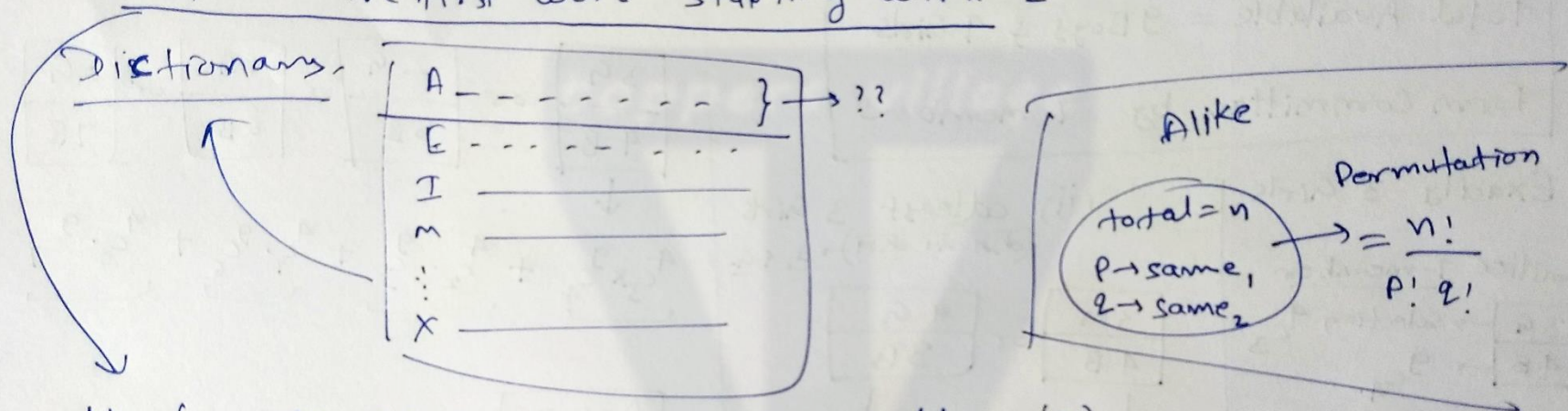
$${}^4C_3 \times {}^9C_4 + {}^4C_2 \times {}^9C_5 + {}^4C_1 \times {}^9C_6 + {}^4C_0 \times {}^9C_7$$

Answer

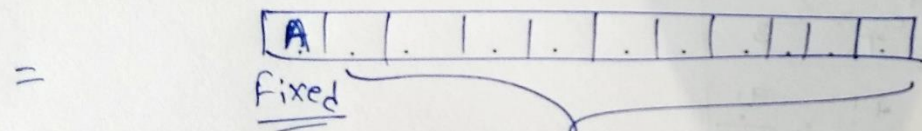
Q.4 'EXAMINATION'

In alphabetical order $\rightarrow$ AA E II M NN O T X $\rightarrow$ total letters = 11

Before the first word starting with 'E'



No. of words starting ~~the~~ with the letter 'A'



Restriction
Priority

= $\frac{10!}{2! \cdot 2!}$

10 Seats.

A, E, II, M, NN, O, T, X

alike

alike

$$\begin{array}{r} 126 \\ \times 72 \\ \hline 252 \\ 882 \times \\ \hline 9072 \end{array}$$

= $\frac{10 \cdot \overbrace{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5}^{10} \cdot \cancel{4} \cdot 3 \cdot 2 \cdot 1}{2 \times 2}$

= $10 \times 10 \times \underline{72} \times \overbrace{42}^{126} \times 3 = 907200$

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Miscellaneous Exercise on chapter ⑦

⑤. Available = 0, 1, 3, 5, 7, 9

6-Digit No.

"Divisible" by 10

Restriction (Priority)

Fixed, 0

Priority

5 · 4 · 3 · 2 · 1

= 5 × 4 × 3 × 2 × 1

= 5!

(Last Digit = 0)

No. of ways = 120

⑥

Available

5 Vowels

21 Consonants

Select

Desired

2 Vowels

2 Consonants

only selection

$5C_2$

$\times$

$21C_2$

Arrange

AE R T -

R A T E -

After Selection Available → 4 letters

(2V, 2C)

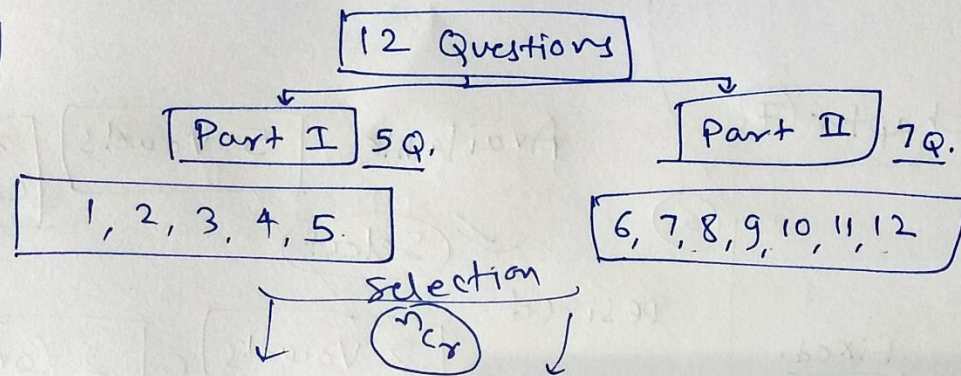
4 · 3 · 2 · 1

Total No. of words =

$\frac{4 \times 3 \times 2 \times 1}{\text{Arrange}} \times \frac{5C_2 \cdot 21C_2}{\text{Selection}}$

= 4 · 3 · 2 · 1 · $\frac{5!}{3!2!} \times \frac{21!}{19!2!}$

Q. 7



A Hempt
8
Questions

at least 3
from each Part

Case-I.

or

Case-II.

or

Case-III.

3 Q (I) & 5 Q (II)

4 Q (I) & 4 Q (II)

5 Q (I) & 3 Q (II)

$$= \left[{}^5C_3 \times {}^7C_5 \right]$$

$$+ \left[{}^5C_4 \times {}^7C_4 \right]$$

$$+ \left[{}^5C_5 \times {}^7C_3 \right]$$

= Answer

$$nCr = \frac{n!}{(n-r)! r!}$$

✓

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Miscellaneous Exercise on Chapter 7

Q. 8

52 Cards Total $\xrightarrow{\text{Select}}$ 5 Cards

4 Kings & 48 Others

1 King & 4 other Cards

Select.

Exactly

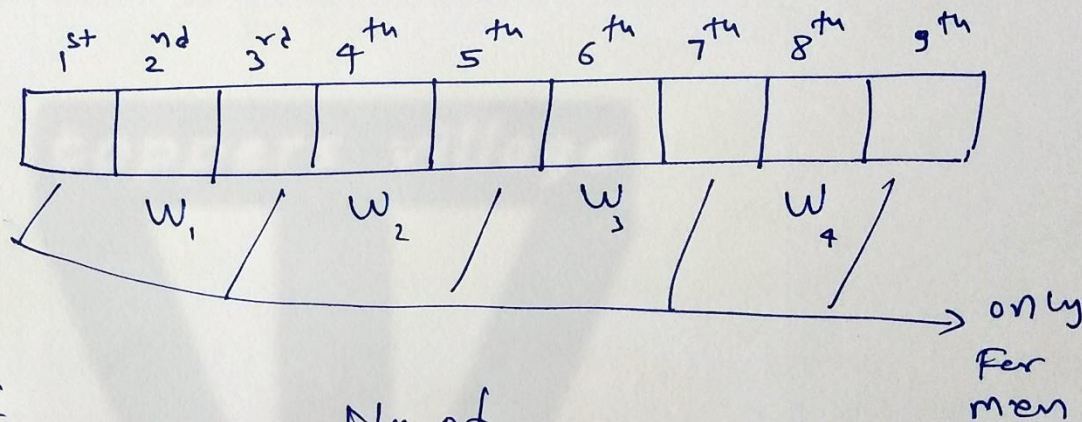
$$\begin{aligned}\text{No. of ways} &= {}^4C_1 \times {}^{48}C_4 \\ &= \text{Answer.}\end{aligned}$$

$${}^nC_r = \frac{n!}{(n-r)! r!}$$

[Q.9] Available $\rightarrow$ 5 men, 4 women

Women occupy Even places

Total seats = 9



$\text{No. of ways} = \text{No. of Arrangements of women}$

$$= \begin{matrix} \square_{2^{\text{nd}}} & \square_{4^{\text{th}}} & \square_{6^{\text{th}}} & \square_{8^{\text{th}}} \end{matrix} = (4 \times 3 \times 2 \times 1)$$

$$= 4! \times 5!$$

$$= 24 \times 120$$

$$= 2880$$

$\text{No. of Arrangements of men}$

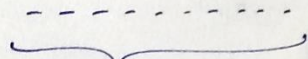
$$\begin{matrix} \square_{1^{\text{st}}} & \square_{3^{\text{rd}}} & \square_{5^{\text{th}}} & \square_{7^{\text{th}}} & \square_{9^{\text{th}}} \end{matrix} = (5 \times 4 \times 3 \times 2 \times 1)$$

Q.10

25 Students



Special '3'

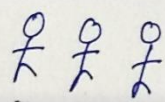


22 others

Party 10 Students

Case I

Others



Either all of them

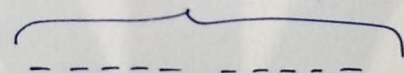
$$= {}^3C_3 \times {}^{22}C_7$$

$$= 1 \times {}^{22}C_7$$

+

Case II

all 10 other



or none of them

$$= {}^{22}C_{10}$$

+

$$nCr = \frac{n!}{(n-r)! r!}$$

$$\text{Total No. of ways} = {}^{22}C_7 + {}^{22}C_{10}$$

$$= \frac{22!}{15! 7!} + \frac{22!}{12! 10!}$$

Ans.

[Q.11] ASSASSINATION $\rightarrow$ AAA, II, NN, O, T, SSSS,

All the S's are
together

Bag method,

AAA, II, NN, O, T, SSSS

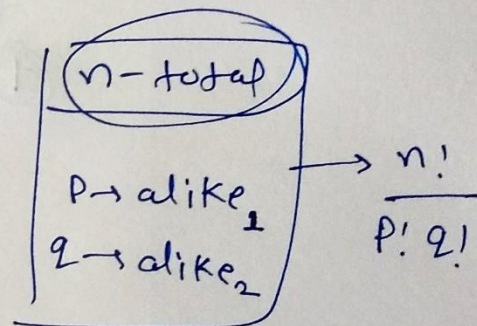
10 objects $\rightarrow$ Permutations
No. of words = $\frac{10!}{3! 2! 2!}$

AAA II NN

$$= \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 2 \times 1 \times 2 \times 1}$$

$$= 10 \times 10 \times 72 \times 21$$

$$= 151200$$



$$\begin{array}{r} 72 \\ \times 21 \\ \hline 144 \\ \times 1512 \\ \hline \end{array}$$

All Useful Links:

- **YouTube Channel : Toppers Village** (Get full and free video lectures of Class 10 & 11 Maths) - <https://www.youtube.com/toppersvillage>
- **Website :** www.toppersvillage.com
- **Instagram :** <https://www.instagram.com/toppersvillage/>
- **Facebook Page:** www.facebook.com/ToppersVillage/
- **Facebook Group:** <https://www.facebook.com/groups/ToppersVillage>